Optimal Aerodynamic Attitude Stabilization of Near-Earth Satellites

R. RAVINDRAN* AND P. C. HUGHES†
Institute for Aerospace Studies, University of Toronto, Canada

A near-Earth satellite orbiting in the altitude range of 150 km to 450 km encounters small but non-negligible aerodynamic forces. An analysis of a system which generates sufficient aerodynamic torques to achieve active attitude control from "all-moving" control surfaces is presented. The satellite configuration considered has four control surfaces and the dominant gravity gradient and aerodynamic torques are included in the analysis. The equations of motion are linearized and optimal control theory concepts are applied. A feedback control system is synthesized subject to the constraint of minimum drag. Numerical studies indicate that aerodynamic surfaces of reasonable size are achievable. Damping times of the order of from a few orbits to a fraction of an orbit are possible.

Nomenclature

A, B, C = moments of inertia of satellite about roll, pitch, yaw axes
A_i = area of control surface i
$\mathbf{b}_i$ = center of pressure of 8th control surface with respect to satellite mass center
C_i = $\cos\theta_i$ ($i = 1, 2, 3$)
C_{pi}, $C_{\tau i}$, C_{Di} = normal force, shear force, and drag coefficients for control surface i
C_{m_Λ} = body moment coefficient derivative with respect to body angle of attack
D = drag
$\mathbf{G}$ = torques (subscripts: *D*–disturbance, *C*–control, *G*–gravitational, *A*–aerodynamic, *CB*–center body, *CS*–control surfaces)
h = altitude
$\mathbf{I}$ = inertia matrix for satellite ($\mathbf{1}$ = unit matrix)
i = orbital inclination
$\mathbf{j}_1$ = unit vector along vehicle roll axis
$\mathbf{L}$ = transformation matrix given by Eq. (4)
L = characteristic length of center body.
l_1, l_2 = moment arm of control surface with respect to satellite mass center
$\mathbf{n}_i$ = unit vector normal to control surface i
$\mathbf{p}_i$ = normal force on control surface i
R = orbital radius
S_i = $\sin\theta_i$ ($i = 1, 2, 3$)
S = molecular speed ratio
S_A = characteristic cross-sectional area of center body
$\mathbf{V}_R$ = "air" velocity with respect to satellite
$\mathbf{v}_R$ = $\mathbf{V}_R/V_R$ (to first order in $\omega_E R/V$)
V = orbital speed
α_i = angle of attack of control surface i
η = orbital anomaly
$\theta_1, \theta_2, \theta_3$ = roll, pitch, yaw angles
θ_i^c = control angle for control surface i
$\kappa_1, \kappa_2, \kappa_3$ = direction cosines of local vertical with respect to spacecraft axes
Λ = center body angle of attack
μ = gravitational constant for Earth
ρ = atmospheric density
σ, σ' = tangential and normal accommodation coefficients
τ_i = aerodynamic shear force on control surface
$\boldsymbol{\omega}$ = angular velocity of spacecraft with respect to non-rotating (inertial) frame
ω_E = angular speed of Earth
ω_0 = orbital angular speed
ω = orbital argument of perigee

Received November 22, 1971; revision received March 2, 1972. This work was sponsored in part by USAF Grant AF-AFOSR 68-1490, and in part by the National Research Council under Grant A-4183.

Index category: Spacecraft Attitude Dynamics and Control.

* Graduate Student, presently Systems Analyst, Spar Aerospace Products Ltd., Toronto, Canada.

† Associate Professor. Associate AIAA.

Introduction

MANY schemes have been devised to take advantage of some naturally occurring environmental torque for satellite attitude control. In particular, the possibility of stabilization by aerodynamic means has been studied by several authors. For example, Schrello[1] conducted an extensive study of the passive stabilization of near-Earth satellites. Sarychev[2] studied the stability of a satellite with an aerogyroscopic stabilization system, and an early discussion is due to Wall.[3] The combined influence of aerodynamic and gravitational torques on the stability of a passive spinning satellite was studied by Meirovitch and Wallace[4] using the direct method of Liapanov.

This paper is concerned with orbits below 500 km altitude where aerodynamic torques are important; they become dominant below 300 km. Gravity gradient torques may have comparable magnitude. Other torques are either inherently small or under the control of the designer. Consequently the possibility exists of using aerodynamic torques for the attitude stabilization of near-Earth satellites.

An Earth-oriented satellite is analysed. The configuration (Fig. 1) consists of a center body with a long axis of symmetry (nominally aligned with the velocity vector), and cruciform control surfaces mounted as shown. Each is an "all-moving" control surface capable of being rotated about its centroidal axis lying normal to the axis of symmetry of the center body. The control surfaces are to be of light weight construction and of suitable area so as to provide sufficient aerodynamic torque

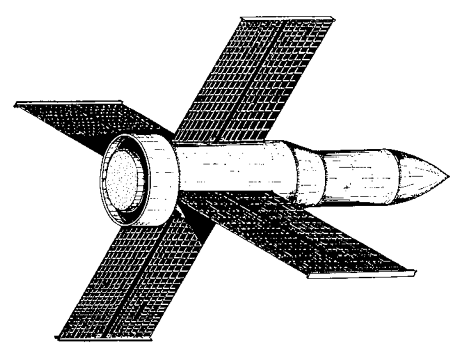

Fig. 1 Configuration (schematic only).

while producing negligible inertial torques due to their angular accelerations.

Several assumptions have been made to simplify the subsequent analysis. These are: 1) The vehicle is in a circular orbit. 2) The time rates of change of the orbital parameters (e.g., due to Earth's oblateness and atmospheric effects) are small in comparison with the orbital rate of the satellite. 3) The isodensity contours are geocentric spheres and the density at any orbital radius is taken from the 1962 U.S. Standard Atmosphere. However, the effect of diurnal density variations on a control synthesized with this assumption is investigated subsequently by introducing a periodic variation in density at any given altitude. 4) Small deflections due to panel flexibility are not considered. 5) The control panels are of light weight construction and hence the moments of inertia of the control panels are assumed to be small to enable one to neglect the variation in the moments of inertia of the satellite and the inertia effects arising from the rotation of the control panels. 6) Only aerodynamic and gravity-gradient torques are considered. 7) Since the external disturbances acting on the satellite are small, the excursions in the state of the satellite from the nominal state are small and consequently a linear analysis is appropriate. 8) The atmosphere is assumed to be rotating at the same angular velocity as the Earth.

Equations of Motion

The rotational dynamics of a rigid satellite can be represented by Euler's equations:

$$(d/dt)(\mathbf{I}\cdot\boldsymbol{\omega}) + \boldsymbol{\omega}\times\mathbf{I}\cdot\boldsymbol{\omega} = \mathbf{G}_D + \mathbf{G}_C \tag{1}$$

For Earth-pointing satellites, the vehicles angular motions may be conveniently referred to a set of rotating orbital coordinates (Fig. 2) whose motion is completely determined by the orbital motion of the vehicle's mass center. The satellite principal axes are oriented with respect to these orbital coordinates by three rotation angles: θ_1 about axis 1, θ_2 about axis 2, and θ_3 about axis 3. The angular velocity of the vehicle relative to inertial space is then expressed in terms of the angular rates of the orbital coordinates, the three orientation angles, and their time derivatives. Since the angular rates of the orbital coordinates are small, except for the rate of change of the true anomaly of the satellite, the angular velocity of the satellite relative to inertial space, expressed in the body-fixed principal axis coordinate system, is given by

$$\boldsymbol{\omega} = \begin{pmatrix} S_3\dot\theta_2 + C_3C_2\dot\theta_1 - (S_1S_2C_3 + C_1S_3)\dot\eta \\ C_3\dot\theta_2 - S_3C_2\dot\theta_1 - (C_1C_3 - S_1S_2S_3)\dot\eta \\ \dot\theta_3 + S_2 \quad \dot\theta_1 + S_1C_2\dot\eta \end{pmatrix} \tag{2}$$

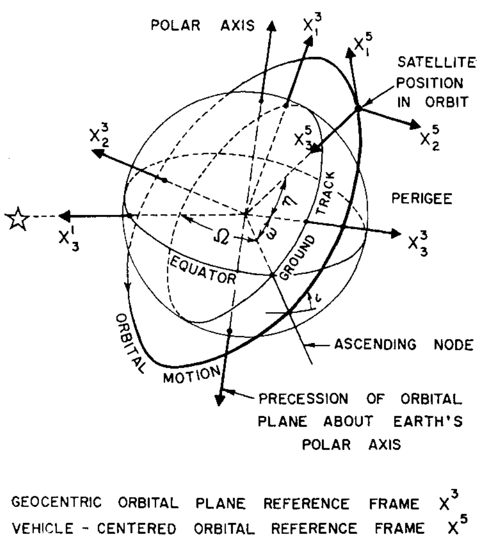

Fig. 2 Reference frames.

The angular acceleration $\dot{\boldsymbol{\omega}}$ can be obtained from Eq. (2) by differentiation, noting that for circular orbits $\ddot\eta = 0$.

The gravitational torque acting on the satellite is

$$\mathbf{G}_G = \frac{3\mu}{R^3}\begin{pmatrix} \kappa_2\kappa_3 & (C-B) \\ \kappa_3\kappa_1 & (A-C) \\ \kappa_1\kappa_2 & (B-A) \end{pmatrix} \tag{3}$$

In fact $\boldsymbol{\kappa} = \mathbf{L}(0 \quad 0 \quad 1)^T$ where $\mathbf{L}$ is the transformation matrix from the orbital coordinate system to the principal axes coordinate system and is given by

$$\mathbf{L} = \begin{pmatrix} C_2C_3 & S_1S_2C_3 + C_1S_3 & S_1S_3 - C_1S_2C_3 \\ -C_2S_3 & C_1C_3 - S_1S_2S_3 & S_1C_3 + C_1S_2S_3 \\ S_2 & -S_1C_2 & -C_1C_2 \end{pmatrix} \tag{4}$$

The relative velocity of the atmosphere with respect to the satellite in the satellite principal axes coordinate system can be represented by

$$\mathbf{V}_R = V[1 - (\omega_E R/V)\cos i]\mathbf{v}_R \tag{5}$$

where the vector

$$\mathbf{v}_R = \mathbf{L}\begin{pmatrix} -1 \\ (\omega_E R/V)\sin i\cos\eta \\ 0 \end{pmatrix} \tag{6}$$

The control panels can be considered as flat plates of negligible thickness. The normal force $\mathbf{p}_i$ and the shear $\boldsymbol{\tau}_i$ acting on the ith control panel are

$$\mathbf{p}_i = \tfrac{1}{2}\rho V_R^2 A_i C_{p_i}\mathbf{n}_i$$

$$\boldsymbol{\tau}_i = \tfrac{1}{2}\rho V_R^2 A_i C_{\tau_i}\mathbf{t}_i$$

where $\cos\alpha_i \triangleq (\mathbf{n}_i \cdot \mathbf{v}_R)$ and $\mathbf{t}_i = -\cos\alpha_i\mathbf{n}_i + \operatorname{cosec}\alpha_i\mathbf{v}_R$. For a flat plate exposed on both sides to the flow, the normal, tangential, and drag force coefficients are, respectively, given by[5]

$$C_{p_i} = [2(2-\sigma')/(\pi)^{1/2}S]\cos\alpha_i e^{-S^2\cos^2\alpha_i} + [2(2-\sigma')\cos^2\alpha_i + (2-\sigma')/S^2]\operatorname{erf}(S\cos\alpha_i)$$

$$C_{\tau_i} = [2\sigma/(\pi)^{1/2}S]\sin\alpha_i\{e^{-S^2\cos^2\alpha_i} + (\pi)^{1/2}S\cos\alpha_i \times [\operatorname{erf}(S\cos\alpha_i)]\} \tag{7}$$

$$C_{D_i} = [2/(\pi)^{1/2}S][(2-\sigma'-\sigma)\cos^2\alpha_i + \sigma]e^{-S^2\cos^2\alpha_i} + 2\{(2-\sigma'-\sigma)\cos^3\alpha_i + [(2-\sigma')/2S^2 + \sigma]\cos^3\alpha_i\}\operatorname{erf}(S\cos\alpha_i)$$

The aerodynamic torque acting on the satellite due to the presence of the control panels can now be written:

$$\mathbf{G}_{CS} = \sum_{i=1}^{4}\mathbf{b}_i \times (\mathbf{p}_i + \boldsymbol{\tau}_i) \tag{8}$$

$\mathbf{b}_i$ being the vector representing the position of the center of pressure of the ith control panel with respect to the mass center of the satellite.

The contribution of the center body to the aerodynamic torque acting on the satellite can be approximated by the following expression

$$\mathbf{G}_{CB} = \tfrac{1}{2}\rho V_R^2 S_A L C_{m_\Lambda}\Lambda(\mathbf{v}_R \times \mathbf{j}_1) \tag{9}$$

in which $\Lambda = \cos^{-1}(-\mathbf{v}_R \cdot \mathbf{j}_1)$. Hence, the total aerodynamic torque acting on the satellite is given by

$$\mathbf{G}_A = \mathbf{G}_{CS} + \mathbf{G}_{CB} = \tfrac{1}{2}\rho V_R^2 S_A L\left\{C_{m_\Lambda}\Lambda(\mathbf{v}_R \times \mathbf{j}_1) + \sum_{i=1}^{4} A_i^*\mathbf{b}_i^* \times [\mathbf{n}_i C_{p_i} + (\operatorname{cosec}\alpha_i\mathbf{v}_R - \cot\alpha_i\mathbf{n}_i)]\right\} \tag{10}$$

where $A_i^* = A_i/S_A$ and $\mathbf{b}_i^* = \mathbf{b}_i/L$

The drag experienced by the satellite due to the presence of the control panel is now given:

$$D = \tfrac{1}{2}\rho V_R^2 S_A \sum_{i=1}^{4} A^*{}_i C_{D_i} \tag{11}$$

The drag and the aerodynamic torques due to forces on the control panels depend on the control angles θ_i^c. By varying these four angles the drag and aerodynamic torque can be controlled.

The equations of motion Eq. (1) in nonlinear form can be rearranged as a set of six first-order equations

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}, t) \tag{12}$$

where $\mathbf{x}^T \triangleq (\theta_1\theta_2\theta_3\dot{\theta}_1\dot{\theta}_2\dot{\theta}_3)$ and $\mathbf{u}^T \triangleq (\theta_1^c\theta_2^c\theta_3^c\theta_4^c)$.

Since Eq. (12) will presently be linearized, the explicit form of $\mathbf{f}$ will not be given here in the interests of conciseness. Details may be found elsewhere.[6]

The system of equations represented by Eq. (12) which is to have $\mathbf{x} = \mathbf{0}$ as its equilibrium state, is also to be controlled to remain close to $\mathbf{x} = \mathbf{0}$ in the presence of small perturbations. The control necessary to ensure that $\dot{\mathbf{x}} = \mathbf{0}$ when the satellite is initially at $\mathbf{x} = \mathbf{0}$ (in the absence of all perturbations) is given by

$$\mathbf{f}(\mathbf{0}, \mathbf{u}_0, t) = \mathbf{0} \tag{13}$$

where $\mathbf{u}_0$ is the nominal control.

The drag acting on the satellite control surfaces, as a direct consequence of using aerodynamic control, leads to a reduction in the life of the satellite. Consequently, the nominal control in the present case can be stated as follows. With zero steady-state pointing error and in the absence of other perturbations, maintain the combined aerodynamic and gravitational torques on the satellite at zero; furthermore in the process of so doing minimize the total drag on the satellite. Hence

$$\int_0^{2\pi} D d\eta \bigg|_{\theta=0}$$

will have to be minimized subject to the constraint $(\mathbf{G}_G + \mathbf{G}_A)_{\theta=0} = \mathbf{0}$.

It can be shown that the above calculus of variations problem degenerates to an ordinary minimum problem for each value of η because of the absence of derivatives in the functional. Therefore $D|_{\theta=0}$ has to be minimized subject to the constraint $(\mathbf{G}_G + \mathbf{G}_A)_{\theta=0} = \mathbf{0}$ for all η, $0 \leqslant \eta \leqslant 2\alpha$. The penalty function approach[7] with a first-order gradient method was used to solve the above constrained minimization problem. The results indicate that for equatorial orbits all control angles are zero for all altitudes, as expected. For nonequatorial orbits the nominal control angles are functions of orbital angle, orbital radius, and inclination. Figure 3 shows the nominal control angles for a particular satellite configuration (the parameters are given in Table 1) in a polar orbit at 200 km

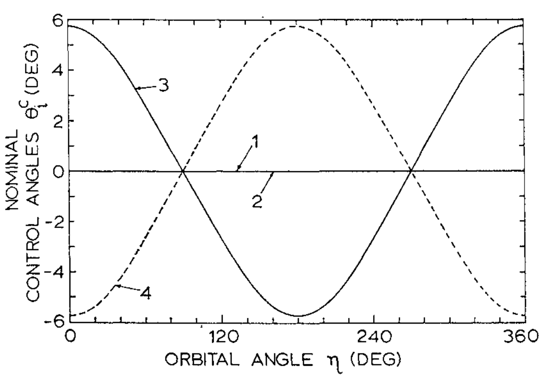

Fig. 3 Nominal control angles for a polar orbit at 200 km altitude.

Table 1 Configuration Parameters

m = 10,000 kg, L = 10m, S_A = 10m²,
l_1 = 5m, l_2 = 2m, A_i = 5m², C_{m_Λ} = 1.0,
A = 30,000 kg-m², $B = C$ = 60,000 kg-m²

altitude. The altitude dependence of the nominal control angles is very slight and the horizontal control panel angles are always zero for nominal control.

Having determined the nominal control the equations of motion given by Eqs. (1) and (2) may be linearized around the nominal state and nominal control. At this point it is convenient to redefine variables:

$$\delta\mathbf{u} = \mathbf{u} - \mathbf{u}_0, \qquad \delta\mathbf{x} = \mathbf{x} - \mathbf{x}_0$$

The independent variable can be changed from t to η using the relation valid for circular orbits, $\eta = \omega_0 t$, $\omega_0 = (\mu/R^3)^{1/2}$. With this notation the linearized equations of motion are expressed as

$$\delta\mathbf{x}' = \mathbf{A}(\mathbf{u}_0(\eta), \eta)\delta\mathbf{x} + \mathbf{B}(\mathbf{u}_0(\eta), \eta)\delta\mathbf{u} \tag{14}$$

where the prime denotes differentiation with respect to η

$$\mathbf{A} = \left(\begin{array}{c|c} \mathbf{0}(3\times 3) & \mathbf{1}(3\times 3) \\ \hline \mathbf{C}(3\times 3) & \mathbf{D}(3\times 3) \end{array}\right)$$

$$\mathbf{B} = \left(\begin{array}{c} \mathbf{0}(3\times 4) \\ \hline \mathbf{E}(3\times 4) \end{array}\right)$$

$$\mathbf{C} = \begin{bmatrix} 4\left(\dfrac{C-B}{A}\right) & 0 & 0 \\ 0 & 3\left(\dfrac{C-A}{B}\right) & 0 \\ 0 & 0 & \left(\dfrac{A-B}{C}\right) \end{bmatrix} + \frac{1}{\omega_0^2}\begin{bmatrix} \dfrac{1}{A} & 0 & 0 \\ 0 & \dfrac{1}{B} & 0 \\ 0 & 0 & \dfrac{1}{C} \end{bmatrix}\left\{\frac{\partial}{\partial \boldsymbol{\theta}}\mathbf{G}_A\right\}^*$$

$$\mathbf{D} = \begin{bmatrix} 0 & 0 & \left(\dfrac{A+C-B}{A}\right) \\ 0 & 0 & 0 \\ \left(\dfrac{B-A-C}{C}\right) & 0 & 0 \end{bmatrix}$$

$$\mathbf{E} = \frac{1}{\omega_0^2}\begin{pmatrix} \dfrac{1}{A} & 0 & 0 \\ 0 & \dfrac{1}{B} & 0 \\ 0 & 0 & \dfrac{1}{C} \end{pmatrix}\left(\frac{\partial}{\partial \boldsymbol{\theta}^c}\mathbf{G}_A\right)^*$$

The asterisk signifies an evaluation at $\mathbf{x} = \mathbf{x}_0$, $\mathbf{u} = \mathbf{u}_0$.

Note that for equatorial orbits $\mathbf{u}_0 = \boldsymbol{\theta}_0^c = \mathbf{0}$, consequently the matrices $\mathbf{A}$ and $\mathbf{B}$ become constant matrices and represent a stationary linear differential system. For nonequatorial orbits $\mathbf{u}_0$ is periodic and hence, $\mathbf{A}$ and $\mathbf{B}$ are periodic matrices with period 2π; thus, Eq. (14) represents a periodic linear differential system.

Synthesis of Feedback Control

Referring to Eq. (14), if $\delta\mathbf{u}$ is set to zero, the system under the nominal control $\mathbf{u}_0$ takes the form

$$\delta\mathbf{x}' = \mathbf{A}(\mathbf{u}_0(\eta), \eta)\delta\mathbf{x} \tag{15}$$

In general the system represented by Eq. (15) is not asymptotically stable as can be verified by studying the eigenvalues of the system when A is constant or the characteristic multipliers when A is periodic. In this section the theory of optimal control[7] is utilized to obtain a feedback control which will provide asymptotic stability.

The system can be represented by

$$\delta\mathbf{x}' = \mathbf{A}(\eta)\delta\mathbf{x} + \mathbf{B}(\eta)\delta\mathbf{u}, \qquad \delta\mathbf{x}(0) = \delta\mathbf{x}_0 \tag{16}$$

The aim is to determine a suitable control $\delta\mathbf{u}(\eta)$ as a function of $\delta\mathbf{x}$ given by $\delta\mathbf{u}(\eta) = \mathbf{f}(\delta\mathbf{x})$ which is optimal in some sense and assures asymptotic stability ($\delta\mathbf{x}(\eta) \to 0$ as $\eta \to \infty$). Minimizing a quadratic performance index of the form

$$\mathbf{J} = \int_0^\infty [\delta\mathbf{x}^T(\eta)\mathbf{Q}(\eta)\delta\mathbf{x}(\eta) + \delta\mathbf{u}^T(\eta)\mathbf{R}(\eta)\delta\mathbf{u}(\eta)]d\eta \tag{17}$$

where $\mathbf{Q}(\eta)$ and $\mathbf{R}(\eta)$ are positive semidefinite and positive definite, respectively, leads to an optimal control which is a linear function of the state and is given by

$$\delta\mathbf{u}(\eta) = -\mathbf{R}^{-1}(\eta)\mathbf{B}^T(\eta)\mathbf{S}(\eta)\delta\mathbf{x}(\eta) \tag{18}$$

$\mathbf{S}(\eta)$ is the solution of the matrix Riccati equation

$$\mathbf{S}' = -\mathbf{S}(\eta)\mathbf{A}(\eta) - \mathbf{A}^T(\eta)\mathbf{S}(\eta) + \mathbf{S}(\eta)\mathbf{B}(\eta)\mathbf{R}^{-1}(\eta)\mathbf{B}^T(\eta)\mathbf{S}(\eta) - \mathbf{Q}(\eta) \tag{19}$$

$$\mathbf{S}(\eta \to \infty) = \mathbf{0}$$

The weighting matrices $\mathbf{Q}(\eta)$ and $\mathbf{R}(\eta)$ are chosen as unit diagonal matrices for convenience. Such a choice also implies that all the control and state variables are equally weighted in the cost function. For alternative choices see, for example, Bryson and Ho.[7] $\delta\mathbf{u}(\eta)$ can be represented as

$$\delta\mathbf{u}(\eta) = -\mathbf{K}(\eta)\delta\mathbf{x}(\eta)$$

where

$$\mathbf{K}(\eta) = \mathbf{R}^{-1}(\eta)\mathbf{B}^T(\eta)\mathbf{S}(\eta) \tag{20}$$

are the feedback gains. Hence, with the optimal control Eq. (20), Eq. (16) leads to

$$\delta\mathbf{x}' = [\mathbf{A}(\eta) - \mathbf{B}(\eta)\mathbf{K}(\eta)]\delta\mathbf{x}, \ \delta\mathbf{x}(0) = \delta\mathbf{x}_0 \tag{21}$$

The above equation represents the feedback controlled system and its stability characteristics can be studied by examining its solutions. For equatorial orbits the system is stationary and consequently S and K are constant matrices. In nonequatorial orbits the system is periodic and the S and K matrices are found to be periodic.

Computational Scheme

The analysis is now applied to a specific satellite configuration whose parameters are shown in Table 1. A schematic of the computations is shown in Fig. 4.

The nominal control angles $\mathbf{u}_0(\eta)$ were determined using the penalty function method. When the nominal control angles are periodic, these may be approximated by a Fourier cosine series. Using the nominal control determined above the matrices $\mathbf{A}(\eta)$ and $\mathbf{B}(\eta)$ are then computed. Subsequently, the transition matrix of the system with the nominal control for one or more orbits is computed and plotted. The results may now be examined to determine the stability properties.

The matrix Riccati equation given by Eq. (19) is integrated backwards with the initial condition $\mathbf{S}(\eta_f) = 0$, with η_f set at

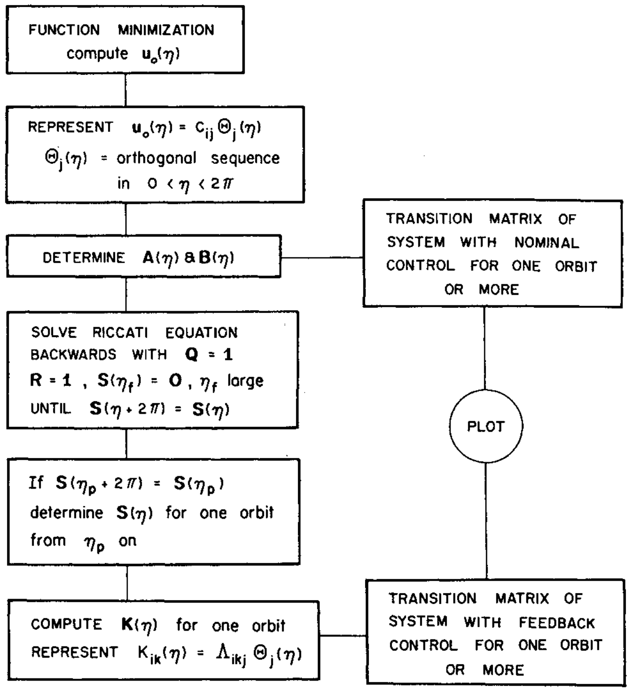

Fig. 4 Computation flow diagram.

3600°. The integration is carried out over four orbits and $\mathbf{K}(\eta)$ is computed from the solution continuously. In the case of equatorial orbits, when $\mathbf{S}(\eta)$ reaches a constant value, the corresponding $\mathbf{K}(\eta)$ value is read out. For nonequatorial orbits, when $\mathbf{S}(\eta)$ reaches a steady periodic state Fourier series are fitted to the elements of $\mathbf{K}(\eta)$ as functions of η over one orbit. The coefficients of the Fourier series are read out along with the norm

$$\sum_{i,j} |S_{ij}(\eta)| \quad \text{of} \quad \mathbf{S}(\eta)$$

After determining the feedback gain matrix $\mathbf{K}(\eta)$, the transition matrix is computed for one or more orbits to study the stability characteristics of the feedback controlled system.

Results

Computations were carried out for the satellite configuration at 200 km and 300 km altitudes. A plot of the nominal control angles for the 200 km polar orbit is shown in Fig. 3. It is seen that the horizontal control panel angles remain zero and the vertical control panel angles vary periodically. Physically, the cross wind present due to the rotation of the atmosphere introduces a periodic yawing moment on the satellite center body and this moment is balanced by creating an opposing yawing moment by suitably rotating the vertical control panels. The nominal control panel angles are all zero for equatorial orbits since in an equatorial orbit the crosswind is zero. Inasmuch as the density appears as a multiplier in the expressions for both the drag and the constraints, the nominal control angles are not affected by local density variations. The results indicate that the altitude dependence of the nominal control angles is very slight. Consequently even in the case of nonequatorial orbits the nominal control need be altered only at discrete altitudes which may be separated by several tens of kilometres.

Response results were obtained for the satellite with nominal control and with feedback control. Figure 5 shows the response of the satellite to initial conditions with nominal control and with feedback control in a 200 km equatorial orbit. The norm of the S-matrix is also shown. It can be seen that the satellite is unstable with nominal control alone whereas with

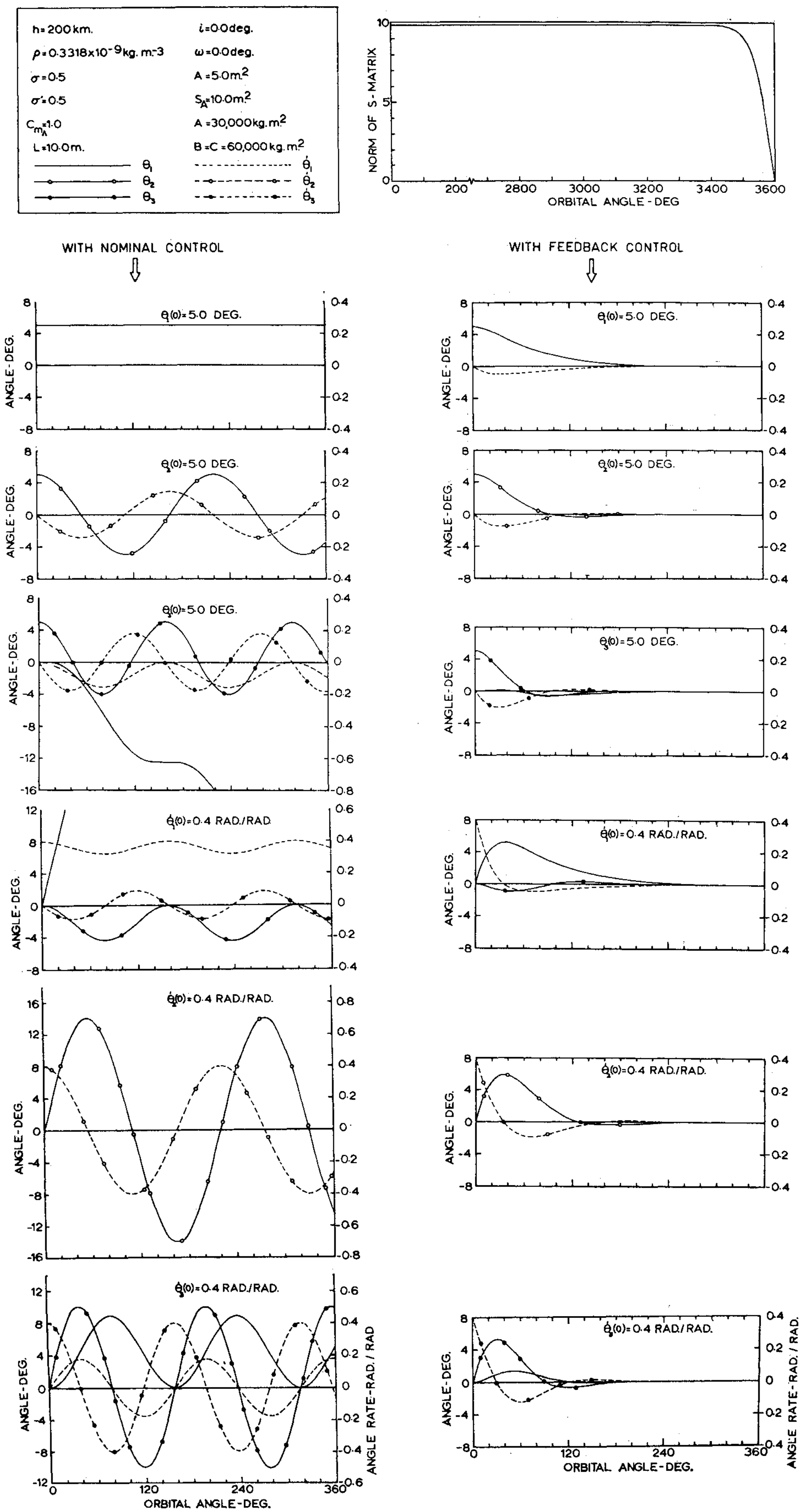

Fig. 5 Performance in a 200 km equatorial orbit: response of satellite to initial disturbances.

feedback control all the modes of the satellites rotational motion are damped out in approximately $\frac{1}{2}$ orbit. Figure 6 shows similar results for a 200 km polar orbit. The plot of the norm of the S-matrix clearly shows its periodic nature. The response of the satellite in polar orbit with feedback control seems to be almost the same as in equatorial orbit. The results for 300 km equatorial orbits and polar orbits were similar to those obtained for 200 km orbits and indicate that the satellite rotational motion dissipates in about one orbit. In other words, the satellite attitude control system becomes less effective with increasing altitude. This is to be expected since disturbing gravity gradient terms decrease more slowly with altitude than the aerodynamic control terms.

The above results correspond to certain fixed values of parameters some of which are left to the choice of the designer (C_{m_Λ}, A, S_A, L, A, B, C, l_1 and l_2) and some others (ρ, σ, σ') which are not known to the designer a priori. It is of interest to study the effect of operating the satellite at off-design values of orbital inclination, orbital altitude, and aerodynamic accommodation coefficients, and to determine the significance of the diurnal bulge.

Suppose equatorial feedback matrices are used for polar orbits. The responses obtained do not differ much from the results obtained using the proper (polar orbit) matrices. Thus it can be concluded that the constant **K**-matrix computed for the equatorial orbit at a given altitude may be used for non-equatorial orbits at the same altitude although the nominal control has to be the one computed for the particular orbital inclination in question.

Because of the orbital decay of the satellite, the orbital altitude continually changes. Hence, theoretically the feedback gains must be computed as a function of altitude and updated continuously. Such an approach is impractical since it introduces complications in the implementation of the control and requires a large amount of computation. Alternatively a form of gain scheduling scheme in which the feedback gains are changed at discrete altitudes separated by several tens of kilometers may be adopted. For such a scheme to be practical the system should perform reasonably well, though not optimally, at off-design altitudes. The effect of using 300 km feedback matrices for 200 km orbits is thus investigated. The results indicate that the performance in the roll mode is not as good as would obtain with the proper 200 km **K**-matrix. In other modes the performance is about the same. On the other hand since the 300 km gain values are rather high, even though the damping times remain reasonably good, the permissible initial disturbances from which acquisition is possible is reduced sharply from the values possible with the use of 200 km gain matrix. On the contrary, these permissible initial disturbances are certainly as large as or even larger than the ones allowable at 300 km. Normally the acquisition problem is important only at the initial altitude, since once having acquired the nominal attitude, the need for reacquiring seldom occurs because large disturbances are rarely encountered. Consequently the feedback control system can be allowed to operate at a lower altitude with the gains computed for some higher altitude.

In computing the response results presented thus far the density has been assumed to be dependent only on altitude. But atmospheric densities are highly time variable at all altitudes in excess of 150 km. The satellite experiences the diurnal density variation with a period approximately equal to the orbital period. Other regular variations in density experienced by the satellite have large time constants. To study the effect of these periodic density variations on the performance of the feedback controlled satellite, a density variation of the form $\rho = \rho_0(1 + k\cos\eta)$, where ρ_0 is the density given by the 1962 U.S. standard atmosphere, and k is the diurnal density variation factor at the given altitude, was introduced. The results obtained for 200 km and 300 km equatorial orbits indicate that variable density does not produce any appreciable change in the performance of the system at these altitudes.

In computing most of the results, the accommodation coefficients were set at $\sigma = \sigma' = 0.5$. Since there is a lack of data to provide accommodation coefficients for typical gases in the atmosphere interacting with various material surfaces at satellite velocities, the above values seem to be reasonable in the light of theoretical estimates. Computations were made to study the effect of these parameters being different from 0.5. The results lead to the conclusion that as long as the **K**-matrix is computed with values of σ and σ' higher than those obtaining in actual operating conditions, the performance of the satellite will be close to the design performance. The physically unrealistic values $\sigma = \sigma' = 0$ would result in the best performance.

Increasing the control panel areas led to a decrease in damping times as well as an appreciable increase in the initial angles and angular rates from which acquisition is possible. Unfortunately, larger control panel areas reduce the lifetime of the satellite. Consequently a suitable control panel area must be selected which will give sufficiently low damping time commensurate with lifetime requirements. Similar improvements in stability properties can be obtained without decreasing the lifetime by suitably increasing the moment arms of the control panels.

Implementation

It was pointed out in earlier sections that the nominal control has a very weak dependence on the altitude. Further, the use of the feedback gain matrix computed for a higher altitude at a lower altitude does not lead to any serious problems in the performance of the system. To avoid computing the nominal and feedback controls continuously as a function of altitude, it is therefore possible to compute and update these at discrete altitudes. This is a form of gain scheduling. It should be noted that in the altitude band between the two altitude values at which the gains are updated the system operates in a sub-optimal fashion. The computations to determine the nominal and feedback control Fourier coefficients may be done onboard if a computer with sufficient capacity is carried. The controller itself requires a certain amount of storage and computational capability to store the Fourier coefficients and to compute the control angles using the Fourier coefficients, state variables, and the orbital anomaly.

Since the density varies considerably with altitude, the above scheme provides good performance at low altitudes and relatively poor performance at higher altitudes. But if the control panel area could be suitably changed with altitude it may be possible to obtain similar performances for a wide range of altitudes. The equations of motion with η as the independent variable are given by Eq. (14). A closer examination of these equations shows that in the absence of aerodynamic effects the motion of the satellite under the action of the gravitational torques alone can be represented by a single set of response curves for all altitudes with η as the independent variable. Or in other words, the transformation used to change the independent variable from t to η happens to be a similarity transformation in the above case, and which eliminates the orbital radius from the equations. But when the aerodynamic effects are present, the orbital radius R is not eliminated from the equation (since $\dot{\eta}$, ρ and V_R are functions of the orbital radius) and hence the response of the satellite is a function of R. The following nondimensional parameters may be defined:

For the center body: $$a_{cb} = \frac{1}{\omega_0^2}\left\{\frac{1}{\text{trace } \mathbf{I}}\right\}(\tfrac{1}{2}\rho V_R^2)SLC_{m_\Lambda}$$

For the control panels: $$a_{cs} = \frac{1}{\omega_0^2}\left\{\frac{1}{\text{trace } \mathbf{I}}\right\}(\tfrac{1}{2}\rho V_R^2)A\,\|\mathbf{b}\|$$

If it were possible to maintain both these parameters constant for all altitudes, then the equations of motion will be independent of R. Of the two parameters a_{cb} cannot be maintained constant. But it is possible to make a_{cb} remain at zero by suitably positioning the center of mass along the satellite's axis of symmetry (i.e., at the aerodynamic center of the main body) thus making C_{m_Λ} zero. Such an approach on the other hand eliminates all the stiffness provided to the system by the moments created by the aerodynamic forces acting on the center body. In contrast to a_{cb}, a_{cs} can easily be maintained constant by varying the control panel areas. The control panel area for any altitude will be given by

$$A = a_{cs}\omega_0{}^2 \text{ trace } \mathbf{I}/(\tfrac{1}{2}\rho V_R{}^2)\|\mathbf{b}\|$$

Note that in the above expression ρ varies by an order of

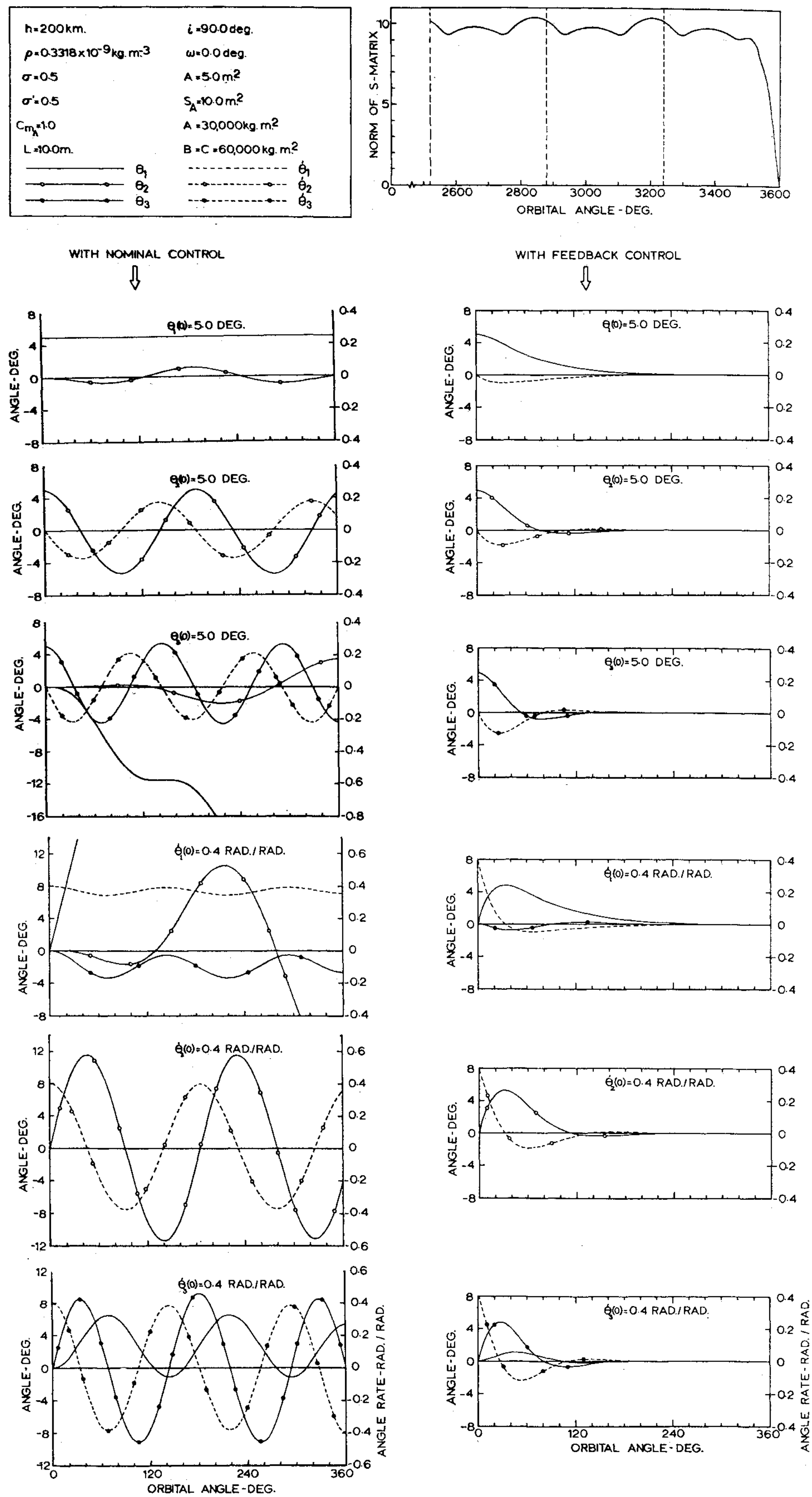

Fig. 6 Performance in a 200 km polar orbit: response of satellite to initial disturbances.

magnitude over the altitude range of interest. But with the currently available technology in deployable and retractable structures it should not be too difficult to vary the control panel areas by orders of magnitude. Such variable area control panels will lead to better performance at higher altitudes.

Conclusions

The performance characteristics and capabilities of various satellite attitude control systems have been compared in Sabroff's paper.[8] Comparison of the present system with other systems on the basis of the various characteristics enunciated[8] is possible only if a fairly complex simulation of the over-all system is carried out. Since this simulation has not been done, only the general range of performance of the present system will be indicated. In the present case stabilization is not possible about an arbitrary attitude whereas it may be possible for mass expulsion systems. Initial attitude angles from which acquisition is possible are fairly large, but the allowable intial rates are relatively small. This compares favorably with most systems except momentum storage/mass expulsion systems. The present system requires attitude sensing and attitude rate information about all three axes. Because of the feedback control scheme utilized, the range of orientation accuracy is expected to be reasonably good. Control cost involves only the power required to rotate the control panels and does not involve any fuel expenditure.

The results of this study indicate that it is possible to exploit the aerodynamic forces acting on a satellite orbiting at certain altitudes to actively control the attitude in an Earth-pointing mode. The density variations due to the diurnal bulge have been shown to be of no serious consequence in devising a suitable feedback control system. Further it has been shown that the extreme variations in density with altitude can be handled by either a gain scheduling scheme or by providing a set of control panels whose areas are varied suitably with altitude. Numerical results indicate that the effect of off-design values of surface accommodation coefficients do not cause any serious problems. Variable gain feedback matrices for nonequatorial orbits have been shown to be unnecessary An evaluation of the effect of the panels on orbit lifetime has been carried out and the results indicate that lifetimes of the order of two to three years are possible.[6]

References

[1] Schrello, D. M., "Dynamic Stability of Aerodynamically Reponsive Satellites," *Journal of the Aerospace Sciences*, Vol. 29, No. 10, Oct. 1962, p. 1145.

[2] Sarychev, V. A., "Stability Conditions of the System of Gravitational Stabilization of Satellites Fitted With Gyro Damping," *Astronautica Acta*, Vol. 14, 1969, pp. 299–310.

[3] Wall, J. K., "The Feasibility of Aerodynamic Attitude Stabilization of a Satellite Vehicle," Paper 787–59, ARS Controllable Satellites Conference, MIT, April 30–May, 1959.

[4] Meirovitch, L. and Wallace, F. B., "On the Effect of Aerodynamic and Gravitational Torques on the Attitude Stability of Satellites," *AIAA Journal*, Vol. 4, No. 12, Dec. 1966, pp. 2196–2202.

[5] Schaaf, S. A. and Talbot, L., "Handbook of Supersonic Aerodynamics: Mechanics of Rarefied Gases," NAVORD Rept. 1488, Vol. 5, Sect. 16, U.S. Government Printing Office, Washington, D.C.

[6] Ravindran, R., "Optimal Aerodynamic Attitude Stabilization of Near Earth Satellites," University of Toronto Institute for Aerospace Studies Rept. 164, Feb. 1971.

[7] Bryson, A. E. and Ho, Y. C., *Applied Optimal Control*, Blaisdell, Waltham, Mass., 1969.

Absolute Stability Analysis of Attitude Control Systems for Large Boosters

S. M. SELTZER*
NASA-George C. Marshall Space Flight Center
Marshall Space Flight Center, Ala.
AND
D. D. ŠILJAK†
University of Santa Clara, Santa Clara, Calif.

A method for performing absolute stability analyses of attitude control systems for large launch vehicles is presented. Absolute stability of these systems is shown in a finite region of the state space. The regions are computed by using the Lur'e-Postnikov Liapunov function. This function is chosen to provide additional information about the exponential property of absolute stability. Significant advantages of the method proposed in this paper are: It is independent of the order of the system; algebraic operations involved in the computations are relatively simple and convenient for machine implementation; and the obtained results are valid not only for a particular nonlinearity but also for an entire class of nonlinear characteristics that satisfy certain general conditions. A system model representing the Saturn V launch vehicle is used to illustrate the method.

Introduction

It will be shown how the attitude control systems for large boosters can be analyzed within the framework of absolute stability. The approach to be developed in this paper opens new avenues for the application of numerous strong results of absolute stability theory[1] to the control of large launch vehicles.

A saturation-type characteristic is used to represent the hydraulic actuators that rotate the gimbaled engines of the boosters. Since the linear part of the system is not stable, the saturation characteristic violates the sector condition, and the absolute stability of the attitude control system can be shown to lie in a finite region of the state space. Regions of absolute stability are estimated by using Liapunov functions of the Lur'e-Postnikov type. An extended version[2,3] of the Popov frequency criterion and the Yakubovich matrix inequalities is used in selecting an appropriate Liapunov function which

Received January 13, 1972, revision received March 24, 1972. The research reported herein was supported in part by NASA under the grant NCR-05-017-010.

Index categories: Launch Vehicle and Controlled Missile Dynamics and Control; Navigation, Control, and Guidance Theory.

* Senior Research Scientist, Astrionics Laboratory. Associate Fellow AIAA.

† Professor, Electrical Engineering Department.